Teacher notes
Topic D

Asteroids attacking Earth!

An asteroid approaches Earth (mass *M* and radius *R*) with speed *u* when far away. The impact parameter (defined in the figure) is *d*. What is the minimum *u* so that the asteroid does not hit Earth? Assume that the speed of Earth does not change appreciably during the encounter and the initial velocity of the asteroid is parallel to that of Earth. If the speed is high the asteroid will move past Earth without hitting it. The worst case scenario is that as the asteroid turns around Earth, the point of closest approach is at a distance $r = R$. The asteroid then grazes the Earth's surface without hitting it. The velocity of the asteroid at the point of closest approach is *v*.

Calculate the minimum value of *u* such that the asteroid does not hit Earth.

Conservation of angular momentum gives: $mud = mvR \Rightarrow v = \frac{d}{R}u$.

Conservation of energy gives:

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$u^2 = \frac{d^2}{R^2}u^2 - \frac{2GM}{R} \quad \text{(substituted } \frac{d}{R}u \text{ for } v\text{)}$$

$$\Rightarrow u = \sqrt{\frac{2GMR}{d^2 - R^2}}$$

If the asteroid speed u is greater than the expression found above, the asteroid will **not** hit Earth.